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THERMAL CRISIS IN THE FLOW FIELD OF A CYLINDRICAL OR SPHERICAL SOURCE

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Variants of the physical phenomenon of thermal crisis in the field of a stationary cylindrical source and a spherical source have been compared in the approximation of an ideal, perfect gas within the framework of Euler equations.

Keywords: *gas, source, sink, energy supply, density, pressure, velocity, Mach number, flow choking, thermal crisis.*

Introduction. Thermal crisis in a one-dimensional flow has been described in [1–3]. In a cylindrical or spherical source [1, 4–6], one can also prescribe the heat supply $g(r)$ per unit mass or unit volume in a certain layer of thickness d. Chemical reactions, electric discharge [7, 8], and laser radiation [9–11] can be the energy source. We investigate the phenomenon of flow choking when the Mach number tends to unity due to the heat supply (thermal crisis).

Formulation of the Problem. The steady-state mass, momentum, and energy equations and the gas equation have the form

$$\frac{d}{dr}(\rho u r^n) = 0\,, \quad \rho u \frac{du}{dr} + \frac{dp}{dr} = 0\,, \quad u\left(\rho \frac{dh}{dr} - \frac{dp}{dr}\right) = g(r)\,, \quad h = \frac{\gamma p}{(\gamma - 1)\rho}\,. \tag{1}$$

Here $g(r)$ is the physical dimensional intensity (rate) of heat release per unit volume: $g(r) = g_0 f(r/r_0)$ [12] or $g(r) = \rho q_0 f(r/r_0)$ [13], where $q_0 f(r/r_0)$ is the intensity of heat release per unit mass of the gas (see also [14]). The quantity g_0 is equal to $\rho_0 q_0 = W_0/r_0^{n+1}$ (W/m^3). The dimensionless distribution function $f(r/r_0)$ is normalized so that the quantity $W_0 = g_0 2^n \pi \int f(r/r_0) r^n dr$ is equal to the total energy-release power: $2^n \pi \int f(r/r_0) r^n dr/r_0^{n+1} = 1$.

In the absence of the external energy supply $g(r) = 0$, the steady-state mass, momentum, and energy equations and the gas equation yield the integral of the source flow rate (strength) m_0, the isentropy condition (p/ρ^γ = const), and the integral of the total enthalpy of the gas flow H_0:

$$2^n \pi r^n \rho u = m_0\,, \quad \frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^\gamma, \quad H = \frac{\gamma p}{(\gamma - 1)\rho} + \frac{u^2}{2} = H_0 \equiv u_0^2 \equiv \frac{\gamma p_0}{(\gamma - 1)\rho_0}\,. \tag{2}$$

Here p_0 and ρ_0 are the maximum values of pressure and density ($u = 0$ for $r \to \infty$, flow to submerged space), $u_0 = [2\gamma p_0/(\gamma - 1)\rho_0]^{1/2}$ is the maximum velocity ($p = 0$, $\rho = 0$ for $r \to \infty$, flow into vacuum). In dimensionless form (the characteristic quantities are velocity u_0, pressure p_0, density ρ_0, and a certain minimum radius r_0), Eqs. (2) and their solutions will be written as

$$u = \frac{m}{r^n \rho}\,, \quad p = \rho^\gamma\,, \quad \rho^{\gamma-1} + \frac{m^2}{r^{2n}\rho^2} = 1\,,$$

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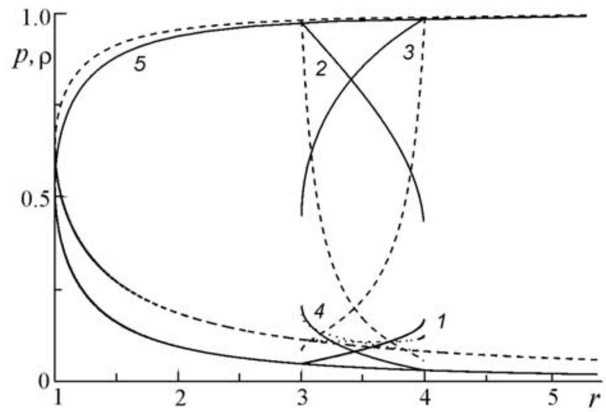


Fig. 1. Distributions of the pressure $p(r)$ (solid curves) and the density $\rho(r)$ (dashed curves) for the E variant: 1) flow into vacuum ($E_{cr} \approx 1.42$), 2) to submerged space ($E_{cr} \approx 18.7$), 3) from submerged space ($E_{cr} \approx 12.5$), 4) from the zone of rarefied gas ($E_{cr} \approx 0.825$), and 5) without heat supply. The cylindrical source (sink) with a uniform heat release $f(r) = C_f = \text{const}$ in the interval $[r_1 = 3,\ r_2 = 4]$, $C_f = C_2 = \frac{1}{\pi}(r_2^2 - r_1^2) = 4.547 \cdot 10^{-2}$. The dimensionless pressure p in fractions of p_0, density ρ in fractions of ρ_0, and coordinate r in fractions of r_0.

$$r = \left(\frac{m^2}{\rho^2 (1 - \rho^{\gamma-1})}\right)^{1/2n}, \quad \mathrm{M}(r) = u \left(\frac{2\rho}{(\gamma - 1)\, p}\right)^{1/2}, \quad m = \frac{m_0}{2^n \pi r_0^n \rho_0 u_0}. \tag{3}$$

Here $\mathrm{M} = u_{\text{phys}}/c_{\text{phys}}$ is the Mach number and $c_{\text{phys}} = (\gamma p_{\text{phys}}/\rho_{\text{phys}})^{1/2}$ is the local velocity of sound. From the minimum condition $dr/d\rho = 0$, we find $\rho(r = 1)$, $p(1)$, $u(1)$, $\mathrm{M}(1)$ and then the explicit expressions for the dimensionless flow rate m and the minimum radius r_0:

$$\rho(1) = \left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)}, \quad p(1) = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}, \quad u(1) = (1 - \rho^{\gamma-1})^{1/2} = \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2},$$

$$m = \rho(1)\, u(1) = \left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} \left(\frac{\gamma-1}{\gamma+1}\right)^{1/2}, \quad r_0 = \left(\frac{m_0}{2^n \pi \rho_0 u_0 m}\right)^{1/n}, \quad \mathrm{M}(1) = 1. \tag{4}$$

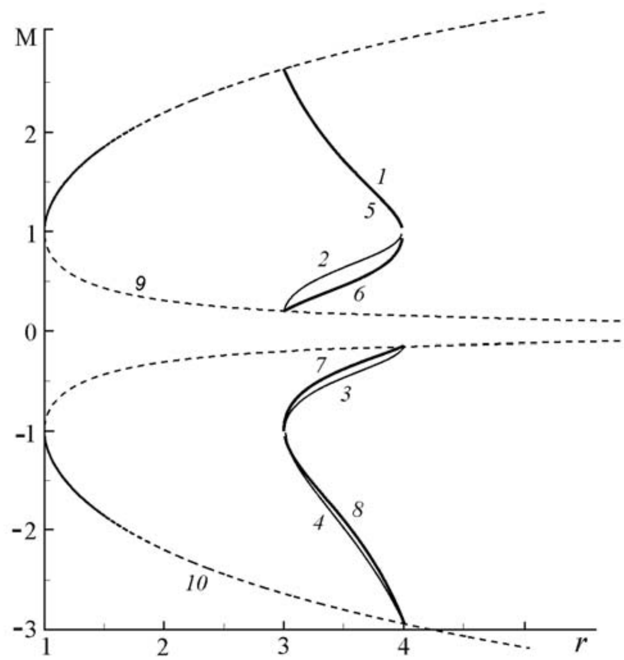
In the minimum-radius cross section, the Mach number is equal to unity, whereas the derivatives of velocity, density, and pressure tend to infinity. Figure 1 gives the solutions $\rho(r)$ and $p(r)$ for the case without heat supply (curves 5); Figs. 2 and 3 give the plots of $\mathrm{M}(r)$ (curves 9 and 10). The quantities u, m, m_0, and M for the sink should be taken with a minus sign.

Flows with Heat Supply. Dimensionless equations with a heat release $f(r)$ prescribed in the interval ($r_1 = 3$, $r_2 = 4$) will be written in the form

$$\frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{u}\frac{du}{dr} + \frac{n}{r} = 0, \quad \rho u \frac{du}{dr} + \frac{\gamma - 1}{2\gamma}\frac{dp}{dr} = 0,$$

$$u\left(\frac{dp}{dr} - \frac{\gamma p}{\rho}\frac{d\rho}{dr}\right) = g(r) \equiv f(r) \times \begin{cases} E \\ Q\rho \end{cases}, \quad E = \frac{(\gamma - 1)\, g_0 r_0}{u_0 p_0}, \quad Q = \frac{(\gamma - 1)\, \rho_0 q_0 r_0}{u_0 p_0}. \tag{5}$$

By virtue of the selection of $g_0 = \rho_0 q_0$ in (1) and for the same values of the parameters of energy supply E and Q, the quantity W_0 in the E variant is the total energy supplied to the gas; in the Q variant, the total supplied energy in



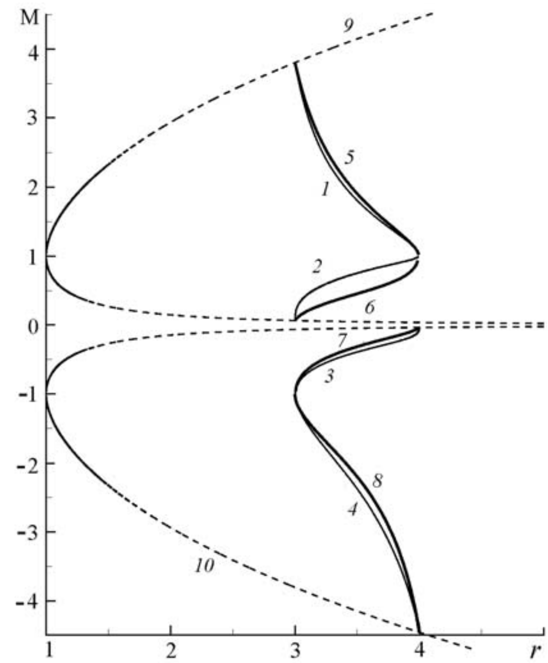


Fig. 2. Mach numbers M(r) for the Q variant (curves 1–4) and the E variant (curves 5–8): 1 and 5) flow into vacuum ($Q_{cr} \approx 13$ and $E_{cr} \approx 1.42$), 2 and 6) to submerged space ($Q_{cr} \approx 123$ and $E_{cr} \approx 18.7$), 3 and 7) sink from submerged space ($Q_{cr} \approx 60$ and $E_{cr} \approx 12.5$), 4 and 8) from the zone of rarefied gas ($Q_{cr} \approx 8$ and $E_{cr} \approx 0.825$), and 9 and 10) without heat supply. The cylindrical source (sink) with a uniform heat release $f(r)$ = const = $C_2 = \frac{1}{\pi}(r_2^2 - r_1^2) = 4.547\cdot10^{-2}$, $r_1 = 3$, $r_2 = 4$; r, fractions of r_0.

Fig. 3. Mach numbers M(r) for the Q variant (curve 1–4) and the E variant (curves 5–8): 1 and 5) flow into vacuum ($Q_{cr} \approx 174$ and $E_{cr} \approx 4.73$), 2 and 6) to submerged space ($Q_{cr} \approx 40{,}600$ and $E_{cr} \approx 565$), 3 and 7) sink from submerged space ($Q_{cr} \approx 11{,}000$ and $E_{cr} \approx 275$), 4 and 8) from the zone of rarefied gas ($Q_{cr} \approx 83$ and $E_{cr} \approx 2.13$), and 9 and 10) without heat supply. The cylindrical source (sink) with a uniform heat release $f(r)$ = const = $C_3 = \frac{3}{4}\pi \times (r_2^3 - r_1^3) = 6.45\cdot10^{-3}$, $r_1 = 3$, $r_2 = 4$; r, fractions of r_0.

the layer $[r_1, r_2]$ is lower than W_0, since the decrease in the density in heating is disregarded in the integral for the quantity W_0.

System (5) for numerical solution will be represented in the form [15]

$$\frac{dp}{dr} = \frac{\dfrac{\gamma p n m^2}{r^{2n+1}} - \begin{pmatrix} E \\ Q\rho \end{pmatrix} f(r)\,\dfrac{\rho m}{r^n}}{\dfrac{\gamma - 1}{2}\,\rho p\,(1 - \mathrm{M}^2)}, \quad \frac{d\rho}{dr} = \frac{\dfrac{\rho n m^2}{r^{2n+1}} - \begin{pmatrix} E \\ Q\rho \end{pmatrix} f(r)\,\dfrac{(\gamma - 1)\,\rho^3 r^n}{2\gamma m}}{\dfrac{\gamma - 1}{2}\,\rho p\,(1 - \mathrm{M}^2)}. \tag{6}$$

The velocity $u(r)$ and the Mach number M(r) are found from (3). The sought critical regimes of choking of the gas flow end in attaining, due to the heat supply, Mach numbers equal to unity at the end of the heat-release zone: at $r = r_2$ for the source or at $r = r_1$ for the sink. The singularity of the solutions $\rho(r)$, $p(r)$, and M(r) in the cross sections r_1 and r_2 in which we have $\mathrm{M} \to 1$ can be attenuated by replacing the coordinate r by another independent variable, $x = |r - r_{1,2}|^{1/k}$, $k = 2, 3, 4, \ldots$.

Figures 1–3 give examples of steady-state solutions with thermal crisis ($\mathrm{M} \to 1$ at the edge of the heat-release zone) for the E and Q variants with a uniform energy supply f = const in cylindrical or spherical layers of unit thickness $d/r_0 = r_2 - r_1 = 1$ in the flow field of the source and the sink at $\gamma = 1.4$ (air). The values of the critical parameters for which we have flow choking are given in the captions to the figures.

The maximum energy release Q_{cr} and E_{cr} for creation of the critical regime of thermal crisis ($M \to 1$ at the edge of the heat-release zone) will be needed for the Q variant of the spherical source to a submerged space, then (in decreasing order of Q_{cr}) for the spherical sink from the submerged space, thereafter for the source flowing out into vacuum, and finally for the sink from a rarefied space. For the E variants, the hierarchy is the same (in decreasing order of E_{cr}): 1) the spherical source to the submerged space; 2) the spherical sink from the submerged space; 3) the source flowing out into vacuum; 4) the source from the rarefied space (from vacuum). In the case of cylindrical symmetry, the hierarchy of flow-choking regimes is preserved in both variants. In the Q variants, the energy requirement is always higher than in the E variants. In the spherical cases the required energy is larger than that in the cylindrical Q and E variants of the source (sink). At $p_0 = 1$ atm, $\rho_0 = 1.225$ kg/m^3, and $r_0 = 1$ m ($u_0 = 760.88$ m/sec, $m_0/2^n\pi = 241$ kg/(s·sr) and 241 kg/(s·m·rad), and $n = 1$ and 2), the total critical power $W_{cr} = E_{cr}u_0p_0r_0^n/(\gamma - 1)$ in the considered examples is approximately 109, 53.0, 0.912, and 0.410 GW for the E variants of the spherical source (sink) and 3.60, 2.413, 0.274, and 0.159 GW/m for the E variants of the cylindrical source (sink) in accordance with the above hierarchy.

When the considered heat-release zone approaches the minimum-radius cross section there can be singularities in the zone and additional constraints imposed on the flow characteristics. Clearly, the characteristics of thermal-crisis regimes, including the singular points, are also dependent on the form of the distribution function of the energy-release source $f(r)$ (e.g., power, exponential, Gaussian, and other forms).

Conclusions. It has been shown that in heat release in a certain layer in the field of a cylindrical or spherical heat source, we have the choking of a gas flow in flowing into vacuum, to a submerged space, from the submerged space, and from the zone of rarefied gas. We have determined the critical values of the parameters of energy supply Q_{cr} and E_{cr} at which the velocity of sound (number $M = 1$) is attained in the closing cross sections of the energy-supply zone for $r = r_2$ (source) or $r = r_1$ (sink) for the variants of the prescribed intensity of heat release per unit volume and per unit mass.

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NOTATION

C_f, C_2, and C_3, normalization constants in the heat-release law $g(r)$ and $f(r)$ for the 2 and 3D cases; d, thickness of the heat-supply layer, m; E, dimensionless parameter of energy supply for the prescribed heat release per unit volume with an intensity $g(r) = g_0f(r)$; E_{cr}, critical value of the energy-release parameter at which the velocity of sound is attained at the end of the heat-supply zone; $f(r)$, dimensionless distribution function of the heat sources; $g(r)$, heat-release intensity (rate), W/m^3; g_0, characteristic intensity of heat release per unit volume, W/m^3; h, enthalpy of the gas, J/kg; H, total enthalpy of the gas, J/kg; H_0, total enthalpy of the stagnant gas, J/kg; k, number, exponent of the auxiliary mathematical quantity x; $m_0 = 2^n\pi r_0^n\rho_0u_0m$, source strength, kg/s; m, dimensionless source strength; M, Mach number; $n = 1$ and 2, in the case of a cylindrical or spherical source; $n + 1 = 1$, 2, and 3, number of measurements of space (one-, two-, and three-dimensional); p, pressure of the gas, Pa; p_0, total pressure (of the stagnant gas), Pa; q_0, characteristic intensity of heat release per unit mass, W/kg; Q, dimensionless energy-supply parameter for the prescribed heat release per unit mass of the gas with an intensity $g(r) = g_0\rho(r)f(r)$; Q_{cr}, critical value of the energy-release parameter at which the velocity of sound is attained at the end of the heat-supply zone; r, coordinate, m; r_0, minimum radius of the mass source (sink), m; r_1 and r_2, dimensionless coordinates of the beginning and end of the heat-release zone; W_0, total power of energy supply, W; u, velocity, m/sec; u_0, maximum gas velocity, m/sec; γ, adiabatic exponent ($\gamma \approx 1.4$ for air); ρ, density of the gas, kg/m^3; ρ_0, density of the stagnant gas, kg/m^3. Subscripts: 0, characteristic and total values of physical quantities; 1 and 2, coordinates of the beginning and end of the heat-release zone; 2 and 3, normalization constants C of the function $f(r)$ for the 2 and 3D cases; phys, physical parameters of the gas; cr, critical values of the parameters.

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